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The Collapse of Cavitation Bubbles and the Pressures thereby Produced against Solid Boundaries

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E. CAVITATION

XX. The collapse of cavitation bubbles and the pressures thereby produced against solid boundaries

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[Plates 48 and 49]

Our object is to present a broad review of this subject as a branch of hydrodynamics, referring both to the well known 'implosion' mechanism first analysed by Lord Rayleigh and, more particularly, to the recently perceived possibility that effects of equally great violence, such as to damage solid boundaries, may arise through the impact of liquid jets formed by collapsing cavities. In §2 a few practical facts about cavitation damage are recalled by way of background, and then in §3 the significance of available theoretical and experimental information about cavity collapse is discussed. The main exposition of new ideas is in §4, which is a review of the factors contributing to shape changes and eventual jet formation by collapsing cavities. Finally, in §5, some new experimental observations on the unsymmetrical collapse of vapour-filled cavities are presented.

1. Introduction

The violent character of vapour cavities in a liquid inasmuch as they can give rise to extremely high hydrodynamic pressures when they collapse is very well known, and since the time of Parsons' and Rayleigh's original commentaries on the matter this property has been generally accepted as the basic explanation for the damage of solid boundaries by cavitating liquids. The mechanism whereby forces large enough to cause damage are brought to bear against a boundary is still rather obscure, however, and both the experimental and theoretical problems remain a long way from being fully resolved. The aim of this contribution is to outline the outstanding questions still posed by this aspect of cavitation and to review some recent findings which may help to answer them.

In particular we focus attention on a question whose importance has probably been suspected by many people since the earliest days of the subject, but which has only recently become well defined. It now appears to be a definite possibility that the impact of liquid jets, formed by involution of collapsing cavities, is a primary factor in cavitation damage, adding considerably to and perhaps often outweighing in its effect the better-known 'implosion' mechanism that Rayleigh (1917) demonstrated theoretically as a source of high pressures. Jet formation in vapour cavities generated by electric sparks in proximity to solid boundaries has been studied by Naudé & Ellis (1961) and by Shutler & Mesler (1965), and their observations vividly establish the possibility now in view; but the evidence so far gained by this particular experimental technique is quite insufficient for any confident assessment of the general role of the jet phenomenon in cavitation damage. An experimental investigation has been begun by us with the object of examining this phenomenon further, and some preliminary results are presented in the final section of this paper.

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2. Some facts about cavitation damage

Our present concern is primarily with the small-scale hydrodynamic processes that may lead to cavitation damage, rather than with the practical facts about the susceptibility of materials to this kind of damage and about the performance of apparatus for erosion tests. These other aspects of the subject are to be taken up by later contributors to the Discussion, and we do not wish to encroach much upon them. To fix this contribution in the practical context, however, it seems desirable to summarize a few more or less generally agreed conclusions on the nature of cavitation damage, such as give particular point to the kind of basic investigation represented in the following pages. For a comprehensive review of the background to current thinking about cavitation damage we may refer to the report by Eisenberg (1963), which includes over a hundred references.

In the past much contention has arisen over the interpretation of cavitation-damage measurements, and undoubtedly the main source of confusion has been that the long-term effects of cavitation attack, as observed for instance in worn hydraulic machinery, are largely brought about by corrosion, which is reinforced in a very complicated way by the prolonged action of the mechanical stresses on the metal surfaces. But ample evidence now exists showing that, at least in its early stages, cavitation damage is essentially mechanical in nature. For example, Plesset & Ellis (1955) examined the initial effects that were caused on various materials by sonically induced cavitation in a chemically inert liquid, specifically toluene under a helium atmosphere, and they found the damage to be practically the same as when water was used.

Inspection of materials exposed even for very short times to intense cavitation reveals localized signs of plastic deformation, leading to fatigue failure. Ductile materials are often found to go through an 'incubation period' during which extensive cold-working may occur but no weight is lost, whereas for brittle materials the rate of weight loss often takes a definite value from the start. On soft materials like aluminium the evidence of damage may be obvious almost immediately upon application of the cavitating liquid, well defined pits being formed in the surface. On hard materials nothing may be seen directly, but use of refined metallographic techniques, such as the observation of X-ray diffraction patterns, has well established that cold-working generally begins with the first exposure to cavitation (see Plesset 1956).

There is now enough definite evidence available to confirm the general conclusion that the effects on a solid boundary are attributable to the collapse of individual cavitation bubbles (see, for example, Knapp 1955, 1958; Hammitt 1963). This fact has for long been commonly assumed, but until quite recently, when the careful experimental observations made by the people just mentioned and others became available, it was a more or less open question whether damage is due to the integrated effect of the profuse cloud of bubbles that is usually present, or whether, as is now generally agreed, just a few members of the cloud have a specially destructive effect.

Thus, the two practical conclusions guiding us are that the essential problem is a mechanical one, and that study of individual collapsing bubbles is the key to understanding the damage process.

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3. General ideas about the collapse of cavitation bubbles

As regards the essential dynamics, the violent action of collapsing cavities was demonstrated very clearly by Rayleigh's well known analysis (1917), which still remains the cornerstone for most theoretical thinking on the subject. In the first place he considered an isolated spherical void collapsing in an incompressible liquid under a constant pressure at infinity, and he showed that as the collapse nears completion the inward velocity of the cavity wall and the pressure inside the liquid become indefinitely large. He recognized, however, that a more realistic physical model is provided by allowing the cavity to contain a small quantity of insoluble gas, whose compression ultimately arrests the inward motion and causes the cavity to 'rebound'. Improved representations of the effects of gas and vapour contents, and various other physical factors, have been included in the theory since Rayleigh's time; but almost all the work done so far on the problem has proceeded on the original assumption of spherical symmetry.

A reason for circumspection about analyses on this basis is that, as was shown by Birkhoff (1954) and Plesset & Mitchell (1956), the spherical form of a collapsing cavity is unstable to small perturbations. Nevertheless, the instability is of a rather weak kind while the inward motion is not being significantly retarded by compression of the cavity contents, and experimental cavities can appear to remain approximately spherical throughout most of their collapse, provided the effects of hydrostatic and other 'environmental' pressure gradients are eliminated (see § 5). But in the concluding stages of collapse, when enormous outward accelerations of the cavity surface occur under the pressure of the contents, the spherical form becomes violently unstable (Benjamin 1954), the cause being essentially the same as the Taylor instability of a plane interface accelerated in the direction from the lighter to denser fluid. For this reason rebounding cavitation bubbles often present a highly irregular, starlike appearance (see, for example, Knapp & Hollander 1948).

In typical conditions the duration of the pressure pulse arising from a single bubble collapse in water is found to be of the order of 1 μs (Ellis 1956), which accords with the common observation that the spectrum of cavitation noise contains appreciable energy well into the megacycle range of frequencies; and there exists fairly conclusive evidence that the maximum pressure at the centre of collapse can reach 10^4 atm (Sutton 1957; Jones & Edwards 1960). When such high pressures arise the compressibility of water is bound to become a vital factor in the motion near the end of collapse, and the pressure pulse radiated from the centre will take the form of a shockwave. Allowance for compressibility of the liquid is therefore a particularly important objective in modern refinements of the Rayleigh theory, and a great deal has already been accomplished in this direction, notably by Gilmore (1952), Flynn (1957, 1964), Mellen (1956), Hunter (1960), Hickling & Plesset (1964) and Ivany (1965). Their work has shown that compressibility has a moderating influence on the collapse of a cavity, in the sense that a large part of the available energy may be stored in a compression wave well before the inward motion is finally arrested, so that the process of concentration of energy towards the centre is less intense than it is in an incompressible liquid. It is found that a shockwave does not form during the collapse, but one may appear soon after a gas-filled cavity begins to rebound, or when an empty cavity finally closes up. The maximum pressure of the compressed contents was shown by

Benjamin (1958) to give a simple general criterion of the capability of a rebounding bubble to generate a true shock, as distinct from a steep but still essentially continuous pressure pulse, and exact calculations by Hickling & Plesset (1964) have demonstrated a commensurate but rather more precise criterion in one particular case. It appears that a shock will be formed at a reasonable distance from a rebounding bubble in water if the internal pressure rises above about 10^3 atm, which is a level evidently exceeded very frequently in typical circumstances of cavitation.

Even when full allowance is made for moderating factors such as those mentioned here, the theory clearly indicates that enormous pressures can be developed near the centre of a symmetrical cavity collapse, and there appears no cause to doubt the experimental estimate that pressures of the order of 10^4 atm are quite typical. This is an impressive magnitude, and the general physical picture that is suggested accounts readily for many of the remarkable effects of intense cavitation such as 'sonoluminescence' and the catastrophic destruction of microbes (Flynn 1964).

It is a popular and quite reasonable interpretation, therefore, to make a loose association of cause and effect between, on the one hand, high collapse pressures developed in the particular way that Rayleigh showed and, on the other, all the familiar manifestations of cavitation damage. However, the view now held by the present writers and several others is that the Rayleigh theory with its modern improvements probably does not tell the whole story as regards the essential factors in cavitation damage, even though it serves very adequately to explain other effects such as cavitation noise (Fitzpatrick & Strasberg 1956; Flynn 1964), specifically those attributable to individual bubbles formed inside a liquid at a reasonable spacing from any solid boundary and from neighbouring bubbles. The high values of internal pressure predicted by the Rayleigh theory are definitely misleading if one loses sight of the fact that they occur only when a cavity has shrunk to extremely small size, and that the amplitude of the radiated pressure wave diminishes with relative radial distance r/R at least as rapidly as $(r/R)^{-1}$. To test the relevance of their spherical-collapse analyses to the damage of solid boundaries, both Hickling & Plesset (1964) and Ivany (1965) estimated some values of peak pressure at distances from the centre of collapse comparable with the initial (maximum) radius of the cavity. Ivany concluded that pressures sufficient to cause damage did not occur at such distances; and while Hickling & Plesset were able to obtain some marginally adequate values, the respective conditions for the cavity were so extreme that the overall model became of very doubtful physical validity (e.g. the initial pressure of the cavity contents had to be made unrealistically small; also the necessary final internal pressure according to this model, when allowance is made for attenuation of the very strong shockwave that is developed, seems unreasonably high—being apparently of the order of 10^6 atm). The point we wish to emphasize is that probably any cavity producing a damaging effect in practice must, throughout its collapse, be so close to the solid boundary that very large departures from spherical symmetry are inevitable;† and this class of situation presents a hydrodynamical problem significantly different and unfortunately much more difficult than the Rayleigh problem.

The focus of our initial conceptions about cavitation bubbles collapsing near a solid boundary lies in the fact that they can in various ways acquire translational motion

† For footnote see facing page.

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towards the boundary. As a result of the collapse, during which the displaced volume may become an extremely small fraction of its maximum value yet its centroid still be brought up close to the boundary, this motion may lead to the delivery of a highly concentrated impulse against the boundary. The principle in view is simply one of momentum conservation, and accordingly the result in view may be regarded separately from the effects of high pressures developed by compression of the cavity contents. Thus, we believe, powerful effects can arise which are *extra* to the Rayleigh implosion mechanism.

The evidence already mentioned about jet formation by collapsing cavities is particularly interesting material for this general interpretation, and we shall take up the matter at some length in §4. Apparently Kornfeld & Suvorov (1944) were first to suggest that the impact of liquid jets could be responsible for cavitation damage, and the possibility was discussed again later by Eisenberg (1950). In the experiments by Naudé & Ellis (1961), which were first to give indisputable evidence of jet formation, bubbles were generated in water by electric sparks between electrodes placed at various distances from a solid wall. Observations on a particular bubble, which at maximum size was roughly spherical but in contact with the wall at about 40° latitude, were found to be in good agreement with the results of an approximate theory; and it appeared that as the volume of the bubble decreased (i.e. when the vapour and gaseous products of the spark had cooled sufficiently for the bubble to be collapsed under the environmental pressure), its surface eventually folded inwards from the pole, finally striking the wall on the axis of symmetry. From studies of the damage produced on a surface of annealed aluminium, it was concluded that pressures caused by such impacts exceeded those arising from compression of the bubble contents.

The more recent experimental study by Shutler & Mesler (1965) was on much the same lines, but they concluded that 'the jet has little or no damage capability'. The limited evidence presented scarcely supports this as a general conclusion, however, and possible reasons for the discrepancy with Naudé & Ellis's findings are easily found. In particular, it seems that the bubbles observed were in a general way less vigorous than those in the previous experiments, probably because the sparks producing them were made by discharging a condenser at considerably lower voltage and so had longer duration for the same total expenditure of energy. In several of the sequences of photographs presented by Shutler & Mesler the spark is seen to have remained alight throughout the complete observed history of the bubble, whereas in the previous experiments the spark was always extinguished before the collapse began.

We shall not discuss the discrepancy any further here because clearly the more important question is the general one whether experiments of this kind provide a true model of the cavitation-damage process in flow systems. It is difficult to estimate the overall importance of the various extraneous factors that the spark technique undoubtedly introduces, and there is the obvious disparity that the bubbles are generated and collapsed *in situ*, whereas

† An exception to this statement must be allowed in the case of a perfectly *hemispherical* cavity founded upon a solid boundary. Cavities with this form can be made experimentally by the electric-spark technique (see, for example, Jones & Edwards 1960); but they are extremely unlikely ever to occur in cavitating flows, and even in sonically generated cavitation, where they might be expected to occur occasionally, no evidence has been found of their being a common event.

real cavitation bubbles generally originate from points far removed from the region of collapse. The velocity of a bubble towards the boundary at the beginning of collapse probably has an important influence on its damage capability, but the method affords no control on this property. It must be acknowledged, therefore, that the experimental modelling of damage-producing cavitation bubbles remains a more or less open subject at the present time.

An experimental fact worth noting in this connexion concerns the rate of occurrence of manifestly destructive cavities, such as to produce individual pits in a test specimen of soft material. Especially in cavitating flows but also in cases of sonically induced cavitation, this is found to ve bery small in comparison with the total rate of formation of cavities in the general neighbourhood of the boundary. For example, from observations on the rate of pitting in an aluminium test section exposed to a cavitation cloud in a water tunnel, Knapp (1955) estimated that only one in 30000 of the transient cavities swept into the region of the test section caused a damaging blow. This fact gives good reason to frame an explanation for cavitation damage as depending on a rather crucial combination of conditions for individual cavities. It may be that conditions are just right (or just wrong, one should perhaps say) when the translational movement of a cavity is just enough to bring the centre of collapse up to the solid boundary, and to cause a jet to form just before the cavity reaches minimum volume and maximum internal pressure; then the jet may have the largest possible velocity, and its impact against the boundary may optimally reinforce the simultaneous effect of the high collapse pressure. As will be discussed later, a cavity may become involuted into toroidal form during contraction, and if this occurs well before the end of collapse the damage potential is probably reduced far below the optimum. Again, the spark-generated cavities observed by Naudé & Ellis and by Shutler & Mesler generally made contact with the solid boundary while they were still much larger than minimum size, and the jets thereafter formed by involution of the far sides of these cavities were comparatively large and low speed ones; accordingly these observations, particularly those by Shutler & Mesler, may not be representative of the most destructive forms of cavity collapse.

4. Theory relating to asymmetric bubble motion

Before presenting our new experimental results, we shall review a few theoretical points that are helpful towards understanding the observed phenomena. The general object is to explain the typical behaviour of collapsing cavities when large departures from spherical symmetry are enforced, either by longitudinal pressure gradients or by proximity to solid boundaries. High speed jets can arise from either of these causes, and so it has seemed reasonable to couple the two closely in our investigation.

A great deal of work on this general subject has been done in connexion with underwater explosion bubbles, and much of the theory, notably the contributions of Sir Geoffrey Taylor, is also valuable in the present connexion. A mine of relevant information is provided in volume II of the compendium, edited by Hartmann & Hill (1950), of wartime reports on underwater-explosion research, and some of the same material can be found in the book by Cole (1948). While pulsating explosion and cavitation bubbles have obvious similarities as regards the essential mechanics, however, there is an important distinction between them in that the typical scale of the collapse, expressed for instance by the ratio of the maximum to minimum volume, is very much larger for cavitation bubbles. At first sight, the effects of hydrostatic and other pressure gradients in the water would appear to be comparatively insignificant for cavitation bubbles, since their size is so much smaller and their life-time so much shorter; but these effects are greatly amplified by the contraction of a cavity, and we shall presently show some direct evidence of their importance in the present case.

The theoretical question central to our discussion is what general effect does a translational movement have on a collapsing cavity? To answer this the momentum of the fluid must somehow come into consideration and consequently one needs to be rather careful—no less than one generally needs to be careful when considering the momentum of a fluid set in motion by the translation of a rigid body. We do not wish to dwell unduly on the basic difficulty, which is anyway quite well known; it has been discussed, for instance, by Lamb (1932, p. 161) and more thoroughly by Birkhoff (1950, p. 158). But in order to develop any secure argument in terms of momentum, one must recognize that the classical perfect-fluid theory poses a paradox inasmuch as the total momentum of an infinite fluid may be indeterminate, notwithstanding that the motion of a finite body through the fluid is related in a simple, definite way to the propulsive forces.

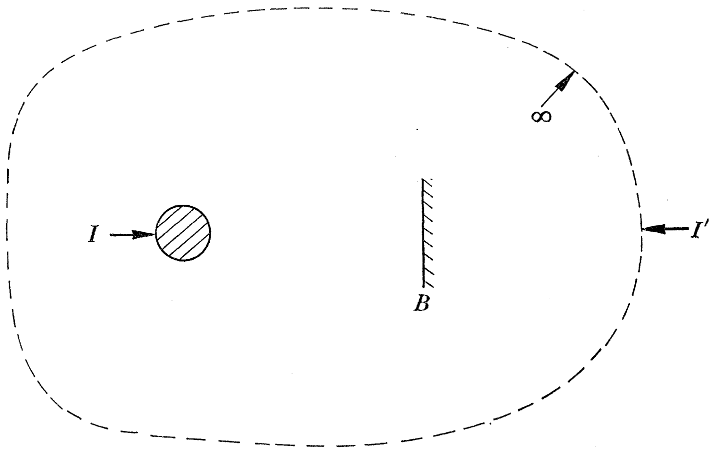

FIGURE 1. Diagram illustrating the discussion of momentum.

The best rationale for the matter is the one originally proposed by Kelvin when he initiated this part of hydrodynamics (see Lamb, ch. 6). The essentials are illustrated in figure 1, where a simply connected body is shown placed alone in an infinite expanse of fluid whose envelope is implied by the dashed line. We consider the impulse I that would have to be applied to the body in order to generate any given motion from rest. (If the body is not symmetrical about an axis in the direction of motion, I must comprise an impulsive wrench, but this case need not concern us here.) The impulse thus defined is *not* in general equal to the momentum of the body and fluid together, because its application will generally produce an impulsive reaction in the fluid at infinity (represented by I' in the figure). This reaction, and hence the total momentum of the fluid, depends on how the infinite envelope of the fluid is specified. For example, if one insists that the fluid is contained in an infinite rigid box, it follows that the momentum must be in the direction opposite to the motion of the body, since the centroid of the fluid is obviously displaced this way; then the reaction at infinity exceeds the part of I due to the fluid. Again, Theodorsen (1941) showed that if one considers the fluid in a huge cylinder, taking its axis in the direction of motion and making its length infinitely greater than its width, then the impulsive reaction at infinity vanishes and the Kelvin impulse I equals the total momentum in this case. But the choice of conditions at infinity is really immaterial, because obviously the actual motion of the body will not depend on what happens at infinite distances away. Thus, to be definite, one should always reason in terms of the Kelvin impulse, not in terms of the fluid momentum whose value is in fact immaterial to the physical problem.

Let F denote the external force acting on the body. (The obvious generalization to a vector force, and resolution of I into orthogonal components, need not delay us here; it will suffice to assume F acts along some specific axis about which the body is symmetrical.) Considering the motion of a rigid body under an external force, Kelvin proved that

$$\mathrm{d}I/\mathrm{d}t = F, \tag{1}$$

and this important formula is the key to the present problem. For, when Kelvin's argument leading to (1) is re-examined, it appears to hold equally well for a deformable body, even one whose volume changes during the motion. (The latter case demands caution, of course, because any volume change produces a net flow at infinity.) Hence a closed cavity is included in the range of application, being equivalent to a deformable solid body with negligible mass, and I is then attributable wholly to the inertial effect of the surrounding fluid.

One useful idea upon which we may draw is, therefore, that a bubble projected through an infinite liquid creates a motion with either a constant Kelvin impulse, in the absence of external force, or an impulse that changes at a rate equal to the force in the direction of motion. The effect of a pressure gradient $\partial\bar{p}/\partial x$ extending over the environment of a bubble whose instantaneous volume is $\mathscr{V}$ may, of course, be interpreted as a force $-(\partial\bar{p}/\partial x)\mathscr{V}$ in the x-direction: for example, the buoyancy force $-\rho g\mathscr{V}$ due to a hydrostatic pressure gradient ρg in the downward vertical direction. As a further point of interpretation, appeal may be made to the complementary definition of the Kelvin impulse, namely that an equal but opposite impulse must be applied over the surface of the bubble to arrest its motion at any instant.

According to these lines of reasoning, the impulse associated with a moving bubble presents much the same intuitive physical picture as the momentum of a rigid projectile in free space, and hence the feasibility of impact effects in the process of cavitation damage is immediately appreciated. To account with any certainty for the case of a bubble moving up to a solid boundary, however, the argument so far developed is clearly inadequate, because (1) holds only for a body infinitely remote from any other boundary of the fluid. Several simple extensions of the argument can be used for this case, and though none of them is entirely free from objection the general conclusion reached seems fairly sound. One way is to consider a bubble started towards a *finite* fixed solid body (the object B in figure 1) which is initially at such a distance away that it does not share significantly in the starting impulse. We assume for simplicity that no external force applies other than that necessary to hold the body B in place. As the bubble moves up to B the Kelvin impulse of the composite system will remain constant; and hence we deduce that there must be a forward impact force against B, because when the motion has been arrested the external reaction keeping the body in place has delivered an impulse equal but opposite, and so cancelling, the original impulse to the bubble.

We go on to consider how the effects of the Kelvin impulse may be concentrated when a bubble is collapsed. For bubbles that become seriously distorted away from spherical form a very difficult analytical problem is presented, and cumbersome approximation methods provide the only way to explicit solutions. A few interesting properties of the possible motions can be inferred by general arguments, however, and to this end we now use some results derived in the appendix to this paper. These results relate to any axi-symmetric body moving in the axial direction through an infinite perfect fluid while at the same time undergoing changes in shape and volume. Letting $x_0(t)$ denote the axial coordinate of the centroid of the displaced volume and $\lambda_n(t)$ a set of parameters determining the form of the body, we have that the kinetic energy of the fluid is expressible in the form

$$T = \tfrac{1}{2}M\dot{x}_0^2 + J\dot{x}_0 + T'. \tag{2}$$

Here M, which is a function of the λ_n alone, is interpretable as the induced mass for a *rigid* body having the form defined instantaneously by the λ_n. The energy T', being independent of x_0 and $\dot{x}_0$, is a property only of the rate of deformation (see equation (A 10) in the appendix). The second term on the right-hand side of (2) includes all products between the axial velocity $\dot{x}_0$ and the generalized velocities $\dot{\lambda}_n$, so that we write

$$J = \sum_n B_n \dot{\lambda}_n. \tag{3}$$

As will be discussed in the appendix, the outstanding point here is that the coefficients B_n all vanish when the body is spherical or ellipsoidal, irrespectively of course of how rapidly its volume is changing, and they are generally small unless the body develops a large skewness in form.

Let us return to the specific case of a cavity, for which (2) expresses the kinetic energy of the whole system, and write

$$V = -\int F\,\mathrm{d}x_0 + p_\infty \mathscr{V} - \int p_i\,\mathrm{d}\mathscr{V} \tag{4}$$

for the potential energy. Here p_∞ is the pressure at infinity, assumed constant, p_i is the pressure inside the cavity, and $\mathscr{V}$ is the cavity volume. Clearly, the last two terms in (4) are not explicitly dependent on x_0, and F is identifiable with the external force as explained hitherto. (The effects of a surface tension γ could be represented by adding to (4) the product of γ and the surface area of the cavity, but this complication seems unwarranted at present. It is desirable to bear in mind, however, that surface tension will always tend to keep the cavity spherical, this property being implicit in the fact that the surface area will increase with any one of the parameters λ_n ($n > 1$, say) measuring deviations from spherical form.) One of the Lagrangian equations for the system is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{x}_0}\right) - \frac{\partial T}{\partial x_0} + \frac{\partial V}{\partial x_0} = 0, \tag{5}$$

which shows that
$$\mathrm{d}(M\dot{x}_0+J)/\mathrm{d}t = F. \tag{6}$$
Thus $M\dot{x}_0+J$ is identified with the Kelvin impulse I.

Unfortunately we cannot draw any simple conclusions about the magnitude of J when the deformation of a cavity becomes large; that is, we cannot then say how the impulse will be shared between the component $M\dot{x}_0$ depending on the instantaneous translation and the component J depending on the deformation rate. But we can still infer a good deal on the basis of the facts about J noted below equation (3). In particular, it appears that the velocity $\dot{x}_0$ must increase if an approximately spherical cavity is contracted by a large amount in all its dimensions; for, recalling general experience with rigid bodies, one can be sure M goes down with any large overall reduction in size, while J will not make much of a contribution to I at least at first. Suppose for simplicity that $F = 0$ during a collapse, or that the collapse occurs in a time so short that the change in $\int F\,\mathrm{d}t$ is not very significant. Then
$$M\dot{x}_0+J = \text{const.}, \tag{7}$$
and it follows that while J remains unimportant the first term in the expression (2) for the kinetic energy increases in inverse proportion to M as the collapse proceeds. Moreover, since J vanishes identically with the deformation rate, $M\dot{x}_0$ would return to its original value if the cavity happened to approach some steady form at the end of collapse; in such a case the kinetic energy of translation clearly could become very much larger than at the start. (Note that J represents in effect the impulse that is self-generated by the cavity in a frame of reference moving with the centroid; that is, it represents the impulse with which the cavity tends to propel itself away from the origin of this inertial frame. From this viewpoint the need for a large skewness in form to make J significant is readily appreciated.)

Now, a basic interpretation of the Rayleigh collapse mechanism is that, as $\mathscr{V}$ decreases, the total energy $T+V$ (which is, of course, constant in the present dissipationless model) converts in kind from an initial potential energy, represented mainly by the second term on the right-hand side of (4), to the kinetic energy of the radial motion, until finally a rapid reconversion to potential energy occurs when the pressure p_i of the compressed contents of the cavity rises to high values and the final term in (4) becomes predominant. But we now see that the translational motion may gain kinetic energy at the expense of the motion relative to the centroid; and indeed the possibility has appeared that the whole energy of collapse (i.e. p_∞ times the initial $\mathscr{V}$, very nearly) could be drawn into the translational motion, provided the impulse I were large enough so that the required volume reduction, making M finally small enough, were still insufficient for a significant fraction of the total energy to be imparted to the cavity contents. This line of reasoning, tentative though it has to be in the absence of complete knowledge about J, supports a general intuitive judgement that the possession of a Kelvin impulse by the system will enfeeble the collapse of a cavity in so far as events might be viewed from the centroid. In the external view, however, the transferred energy may be manifested with physical consequences that appear no less important than those of energy concentration in the cavity contents.

The cavity deformations implied in the foregoing discussion cannot be deduced *a priori* in any simple way, at least not when they become large, but several further considerations of a general kind can be made which illuminate the behaviour typically observed. It was shown first by C. Herring (in a paper included in Hartmann & Hill (1950); see also

Birkhoff & Zarantonello (1957, p. 246)) that if a liquid containing a spherical cavity is suddenly subjected to a uniform force-field such as gravity, or equivalently to a uniform pressure gradient, the cavity remains spherical during its initial motion; that is, the motion comprises a rigid displacement, in the direction from high to low pressure, plus a uniform contraction. (This result is indeed immediately deducible from the fact that $J = 0$ for a sphere.) But when the cavity picks up speed it cannot remain spherical, of course, because the hydrodynamic pressure on a moving sphere is not a constant. In fact it will always tend to flatten broadside to the direction of motion, the effect eventually becoming more pronounced on the rearward side. Note that this behaviour will be represented mainly by terms of the type $-\frac{1}{2}\dot{x}_0^2(\partial M/\partial\lambda_n)$ in the Lagrangian equations for the unsymmetrical components of the motion, rather than terms derived from J. Moreover, the initial tendency to flatten is well explained by the fact that, as common experience shows, $\partial M/\partial\lambda_n$ will be largest for parameters λ_n that measure distortions into oblate form.

For a full appreciation of the typical shape changes observed in moving cavities we may turn to the very extensive studies of them in the instance of underwater-explosion bubbles rising under gravity (see, for example, Cole 1948 or Hartmann & Hill 1950). A point of special interest to us now, which is already strongly implied by the preceding energy considerations and has been well recognized in explosion-bubble studies, is that the characteristic tendencies like rearward flattening will be greatly emphasized if the cavity volume decreases by a large amount, as is typical of cavitation bubbles.

An instructive question to ask oneself with regard to this point is, 'What happens ultimately to the shape of a cavity that is shrunk smoothly down to nothing while moving through an infinite liquid?' According to the principles just explained the translational motion must speed up, of course, but an infinite velocity of translation in the limit appears impossible for a cavity with uniform pressure over its surface. Again, if the liquid remained simply connected as the cavity closed up, the Kelvin impulse would have to vanish, which we cannot allow in the absence of an external retarding force. The only reasonable answer is that the cavity must deform in such a way as to make the liquid multiply connected. Circulation can then appear in the liquid (cf. Lamb 1932, §51), and we are left in the limit with a *vortex system* possessing the original Kelvin impulse. For example, the cavity may take on the form of a torus, which makes the liquid doubly connected, so that a hollow vortex ring is produced. The cavity can then be compressed to indefinitely small size while still preserving the same impulse. And clearly the original cavity must fold in from the back (i.e. be threaded through in the direction of motion) in order to produce a circulation with the right sense (cf. Lamb, §§152, 162).

This nicely explains what has sometimes been observed to happen when a cavitation bubble is collapsed after having acquired a translational motion relative to the liquid in its neighbourhood. A jet forms by involution of the back of the cavity, and something well describable as a hollow vortex ring finally appears. Ellis (1956) observed such behaviour in a bubble collapsed sufficiently slowly (i.e. under a small enough pressure) to be considerably affected by gravity, and a fine example of it has recently been recorded by Ivany (1965). He took high speed ciné photographs of bubbles collapsing in the diverging part of the flow through a Venturi section in a water tunnel, and the observed effects evidently arose in consequence of the strong adverse pressure gradient in this region. Also in support of this general idea we should note the calculations made by Kolodner & Keller (1953), with application to explosion bubbles rising under gravity, which were developed to a stage where the supervention of vortex-ring formation was clearly indicated.

As a general interpretation of the phenomenon of jet penetration through collapsing cavities, one can simply say that the liquid is finding the only possible way to preserve its impulse as the cavity size decreases. Or one can say, perhaps more tellingly, that when the impulse is too large to be manifested by translational movement of the cavity in approximately spherical form, the cavity tends to form a torus because the vortex ring evolved is the only flow capable of manifesting the impulse for indefinitely long under the handicap of the reduced volume displacement available. The crucial factor is the magnitude of the impulse in relation to cavity size; and whereas we are interested here in the enforcement of the effects by large volume reductions, it must be recognized that similar behaviour can ensue when a bubble whose volume remains approximately constant is somehow projected into a liquid with an exceptionally large impulse. We recall that Walters & Davidson (1963) have observed large toroidal bubbles produced by releasing vigorous pulses of air through a tube in the bottom of a water-filled tank.

It remains to consider briefly the theoretical problem of cavities collapsing under the influence of solid boundaries. Perhaps foremost amongst relevant bits of theory there is the very well known result that a pulsating body is attracted towards its 'image' in a rigid plane. In the case of a sphere pulsating with small amplitude this effect can be explained by a neat argument discovered by C. A. Bjerknes nearly a century ago, for which we may refer to Birkhoff & Zarantonello (1957, ch. 11, §5). It is much more difficult, however, to account for the corresponding effect that is observed to occur when a bubble undergoes *large* contractions in proximity to a rigid plane, although considerable progress towards understanding this case has been achieved through analyses based on the assumption that the bubble remains strictly spherical (i.e. its symmetry is imagined to be maintained by kinematical constraints which do no work during the motion). Extensive calculations on these lines were set out in a war time report by M. Shiffman and B. Friedman (included in Hartmann & Hill 1950), and more have been made since then by Green (1957). This work shows clearly that whenever a cavity begins its collapse from a position of rest fairly close to a rigid wall, say, when the nearest distance between the two at the start is about equal to or is less than the cavity radius, large movements of the centroid towards the wall are to be expected. Nevertheless, the constraints on the form of the cavity seriously limit the range of application to cavitation bubbles, even if not so much to explosion bubbles which do not suffer quite such large contractions and consequent distortions from spherical form.

An analysis allowing for shape perturbations was made by Rattray (1951). Unfortunately his method of approximation became unreliable at a degree of deformation that is still fairly small, and though some evidence of jet formation was found this was not very conclusive. The most interesting property established by Rattray's calculations is that an initially spherical cavity collapsing under the influence of a rigid wall at first becomes elongated in the normal direction; it is only at a later stage that the cavity flattens on the side farther from the wall and may go on to develop a jet. This behaviour is presumably accountable to the greater mobility of radial flow in the direction parallel to the wall than in the normal direction. In the calculations made by Naudé & Ellis (1961), which were mentioned in §3, a particular type of jet formation was clearly demonstrated; but for initial conditions they took a stationary cavity already partially in contact with the wall, approximating to a certain experimental cavity generated explosively by an electric spark, and so these calculations are of doubtful relevance to flow-produced cavities.

Apart from the generalized Bjerknes effect, in which the attraction is induced by the volume changes, there are several other effects whereby a cavity might be propelled towards a solid boundary in a flow system. For example, if a cavity is carried by a flow under a tangential pressure gradient along a boundary, its relative motion parallel to the boundary will produce an attraction, since relative velocities in the liquid will be higher and hence pressures lower on the near side. Again, in a low pressure region where cavities originate, pressure gradients normal to the boundary will generally be such as to propel individual cavities towards it, since minima of pressure cannot occur inside the liquid (Lamb 1932, §44). We do not mean to imply, however, that augmentation of the Bjerknes effect will necessarily increase the damage capability of collapsing cavities. Indeed just the opposite may be true because, as we pointed out at the end of §3, too large a normal movement may precipitate jet formation too early in the collapse for it to be most effective, the cavity being folded up into toroidal form before the highest possible velocities and pressures are reached. We can now support this statement by analogy with the simpler case considered earlier in this theoretical discussion; the interpretation of jet formation as an impulse-conserving mechanism certainly carries over in a general sense to the present case, though it must be recognized of course that positive pressures exerted against the boundary finally annul the impulse of an approaching cavity.

An experimental fact perhaps significant in this regard is that cavitation damage induced by flow is often most severe in the neighbourhood of stagnation points downstream from the low pressure zone, notably at the tail end of the large 'fixed' cavity (overlaid by a cloud of small transient bubbles) that may form on a body held in a water-tunnel (cf. Knapp 1955). In such a region the normal pressure gradient will be positive towards the boundary, so tending to retard approaching small bubbles. It seems reasonable to conjecture that optimal conditions for a damaging blow may occur when an occasional bubble is swept up to the boundary with such a speed and size that the collapse concludes against the boundary, and that the effect of the pressure gradient offsets the Bjerknes effect to just the right extent to delay the formation of a jet until the end.

In hard fact, nevertheless, the diversity of factors that may influence the rate at which a cavitation bubble approaches a solid boundary, coupled with those influencing its rate of collapse, debars any comprehensive interpretation in the present state of knowledge; and it must be accepted that the conditions for a maximum damaging effect are still wholly in the realm of conjecture.

5. Experiments

We wish to present a few experimental results illustrating some outstanding features of unsymmetrical cavity collapse. A more complete account of our observations and of the experimental method will be presented in a later paper.

Apparatus and procedure

The central part of the apparatus was a reinforced Perspex box, of $9\frac{1}{4}$ in. square internal horizontal cross section, which in the experiments to be reported was filled with water to a depth of about 10 in. The box was closed by an air-tight lid, and the air space above the water surface could be exhausted by a jet pump; its pressure was generally made about 0·04 atm, the precise value being set by an adjustable leak into the vacuum system.

Before an experiment the water was degassed and rendered capable of withstanding small tensions without cavitating. This was done by putting the water under vacuum and vibrating the box vertically with an amplitude sufficient to produce vigorous and widespread cavitation. (Note that a vertical periodic acceleration with amplitude greater than g will create negative 'hydrostatic' pressures for part of each cycle.) The vibration was continued for about an hour, during which time the noise made by the cavitation became progressively sharper, indicating more violent collapse, and at the end of which the cavitation would cease if the tensions produced in the water were made less than a certain value. Thus the water appeared to have acquired a small but appreciable tensile strength.

In the experiments large vapour-filled cavities were grown from small hydrogen nuclei which were formed by electrolysis on a platinum electrode at the bottom of the box. The electrode was embedded in epoxy resin, with only a minute portion of its surface exposed to the water. Single nuclei with radii down to about 0·1 mm could be formed by passing pulses of current with suitable amplitude and duration through the electrode, and they floated upwards into the required position for generation of the large cavities. The characteristics of the pulse had to be adjusted rather carefully to insure that only a single nucleus broke clear of the electrode each time, and various refinements of technique in making electrodes for this purpose are still under study.

A novel method was used for generating the vapour cavities. When the tiny nucleus had reached the required position, at about the centre of the volume of water, the box was struck downwards by a heavy bar suspended on a spring above it. As a result of the blow the sudden downward acceleration of the box, which was considerably greater than g, produced a large negative hydrostatic pressure in the water, and a cavity consequently opened up from the nucleus. The time for growth of the cavities to maximum size was generally comparable with the time of the subsequent collapse, typically about 0·005 s.

An important feature of the apparatus was a means for creating gravity-free conditions during the collapse of a cavity. The box was mounted on a platform which could slide freely upon two vertical columns, and prior to an observation being made it was suspended by an electromagnet. In the process of striking the box so as to generate a cavity, the downward movement of the bar caused a break in the supply of current to the electromagnet, with the result that the box was in free fall subsequent to the blow. A fall of a few centimetres allowed sufficient time for the observed collapse of the cavity to be free from the effects of gravity.

The cavities were observed by means of high speed ciné photography, using a rotating-drum camera and back-lighting with a flash tube. A sequence of flashes, at a chosen rate and total duration, was initiated by the signal from an accelerometer which was fixed to the box and so responded to the impact of the bar. (Oscillographic records of the acceleration of the box and of the pressure in the water were taken, for comparison with the measured time histories of the cavities, but we shall not discuss these results here.)

Observations on collapse under gravity

Figure 2, plate 48, shows four successive views, at intervals of 2 ms, of a cavity formed near the centre of the water, far away from the walls of the container. The free-fall device was not used in this experiment, and the curious behaviour observed was undoubtedly an effect of gravity.

The first frame shows the cavity at nearly maximum size, with a diameter of 2·4 cm. The second frame shows it having contracted to about half this size, and as in the first frame there appears to have been no significant departure from spherical form. During the interval between the second and third frames, the cavity collapsed down to microscopic size, so that the third and fourth frames show the cavity on its rebound.

The remarkable situation depicted in the third frame may be explained as follows. During its previous history, particularly the time spent near maximum size, the cavity picked up a vertical impulse through the action of gravity (i.e. under the hydrostatic pressure gradient). This was small enough to produce no noticeable effect in the early stages of the collapse, as viewed in the first two frames; but, for the reasons discussed at length in §4, the effect of the impulse became crucial under the enormous reduction of displaced volume in the final stages. A high-speed jet developed during this brief phase, of such vigour that it still persisted when the cavity had rebounded back to large size. The jet can be seen distinctly in the third frame of figure 2, passing upwards through the cavity and entering the liquid above with such force that a conical protrusion of the cavity surface was created behind it. In the fourth frame the jet can still be seen, but it appears to have weakened to the extent that the previous excrescence at the top has been able to close up.

A similar cavity observed in gravity-free conditions showed no vestige of this behaviour, confirming that gravity was the vital factor. Thus the importance of the effects of impulse-conservation, as propounded in §4, is well demonstrated.

Observations on collapse in proximity to a solid boundary

For these experiments a sheet of Perspex was fixed with its plane vertical across the internal span of the box, and cavities were grown from nuclei sited at various small distances from it. The free-fall device was generally used. Examples from a large number of observations made this way are presented in figures 3 to 5, plates 48 and 49. The diameter of the spherical cavity shown in the first frame of figure 3 was 2·2 cm, and the other figures are reproduced to the same scale.

Figure 3 presents a choice of four representative views of a cavity collapsed fairly close to the wall, whose distance from the centre of the cavity at maximum size, as shown in the first frame, was about 1·3 times the radius. (The position of the wall can be made out, on the right-hand sides of the pictures, from the reflexion of the cavity in it. The bead-like features appearing in the cavity surface are merely images of bubbles that formed incidentally a long way behind and for which the main cavity acted as a spherical lens.) The second frame, taken 5 ms after the first, shows the cavity to have become considerably

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elongated in the normal direction as the collapse began. This exemplifies the effect discovered theoretically by Rattray (1951) which was discussed near the end of §4. In the third frame, 9 ms after the first, the cavity is clearly seen to have become involuted on the side farther from the wall; and in the last frame, 10 ms after the first, the jet has passed right through the cavity. From these and neighbouring frames on the ciné film, the jet velocity was estimated to be about 10 m/s. This value was exceptionally low, however, because the jet formed rather early in the collapse. (Note also that raising the environmental pressure to atmospheric, i.e. by a factor of about 25, would scale up all such velocities by 5.)

Figure 4 presents some views of a cavity collapsing further away from the wall. The timing is explained in the legend to the figure. The first frame shows the cavity during its initial growth, the second the cavity at maximum size, and the third and fourth the elongated cavity during the early stages of collapse. The remaining sequence of five frames, taken at 0·2 ms intervals, shows the formation of a jet, concluding with the cavity in a somewhat indistinct but in fact toroidal form.

Figure 5 is included for the sake of several extremely interesting features, although we are unable to explain any of them. In this experiment the cavity was grown from a nucleus sited at about 1·5 mm from the wall, that is, very much closer than in the previous experiments; and it seems possible that the odd effects may have been due at least partially to a vibration of the wall.

The three frames in the top row of figure 5 cover an early phase in the growth of the cavity, before the presence of the wall had any appreciable effect. The second row of frames, covering a phase about 6 ms later, shows the cavity in roughly hemispherical form; and the next row shows it at about the same interval later again, by which time it had developed a curious stratified form. The sequence of six frames in the last two rows covers the final phase of the collapse at intervals of 0·2 ms. A remarkable feature is the bellows-like appearance of the contracting cavity; and perhaps even more remarkable is the unmistakable evidence that a filamentary jet, very much thinner than the cavity at the time, developed in the interval between the first and second frames of this sequence. By measuring the distance traversed by the jet, we estimate its speed to have been at least 35 m/s, and possibly a good deal higher. Hence, when allowance is made for the fivefold increase of speed that would result if the environmental pressure were raised to atmospheric, it may be appreciated that jets like this one could be highly damaging to a solid boundary.

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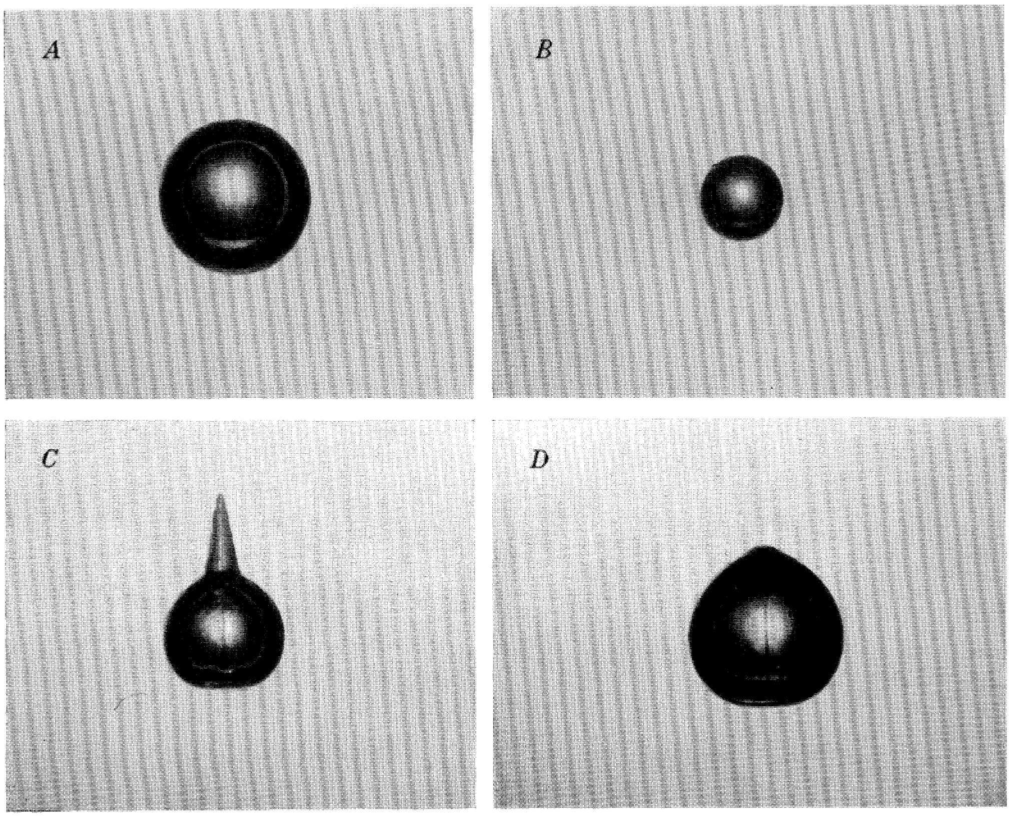

FIGURE 2. Photographs taken during collapse (*A*, *B*) and rebound (*C*, *D*) of cavity far from boundaries of liquid. Interval between frames, 2 ms.

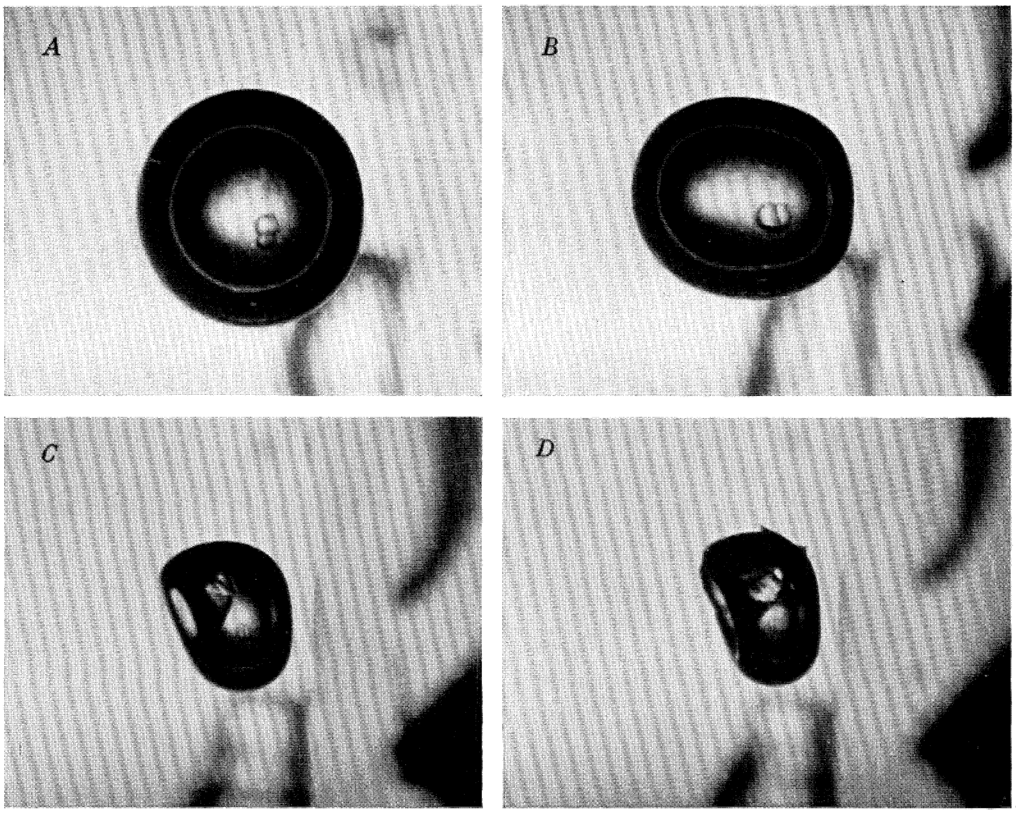

FIGURE 3. Collapse of cavity near a solid wall. Timing: *A*, *B*, *C*, *D* at 0, 5, 9, 10 ms.

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(*Facing p.* 236)

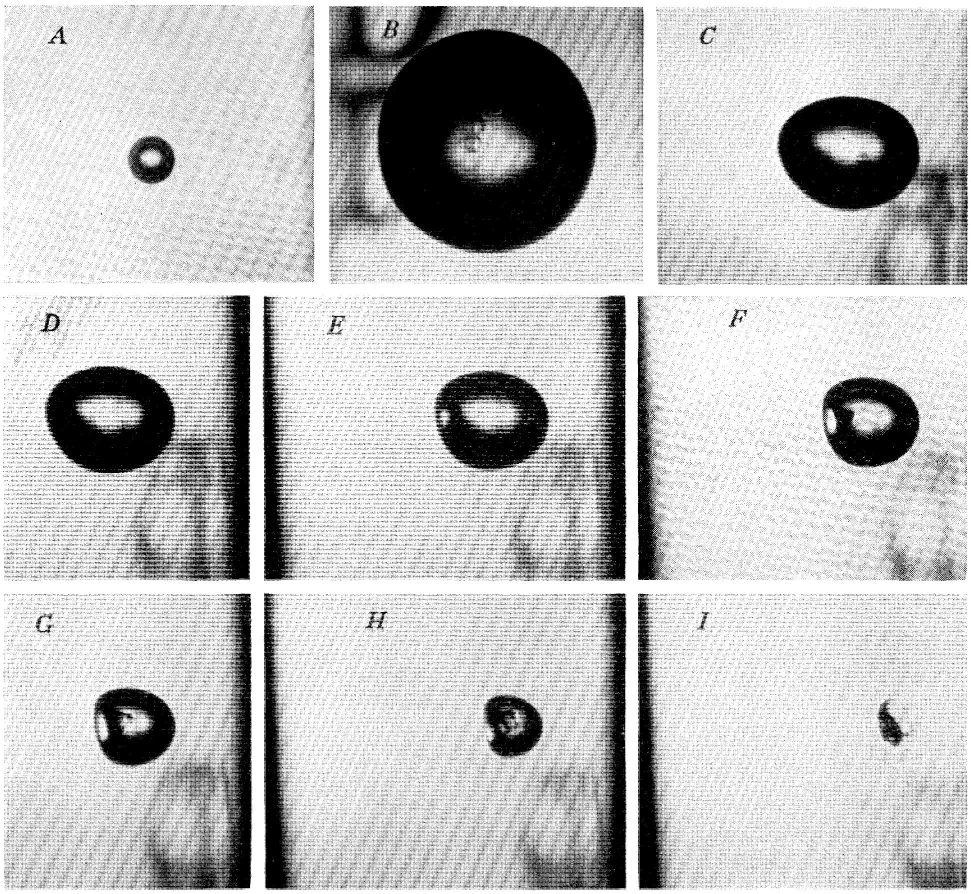

FIGURE 4. Growth and collapse of cavity near a solid wall. Timing: *A*, *B*, *C* at 0, 5·8, 8·8; *D*, *E*, *F* at 9·4, 9·6, 9·8; *G*, *H*, *I* at 10·0, 10·2, 10·4 ms.

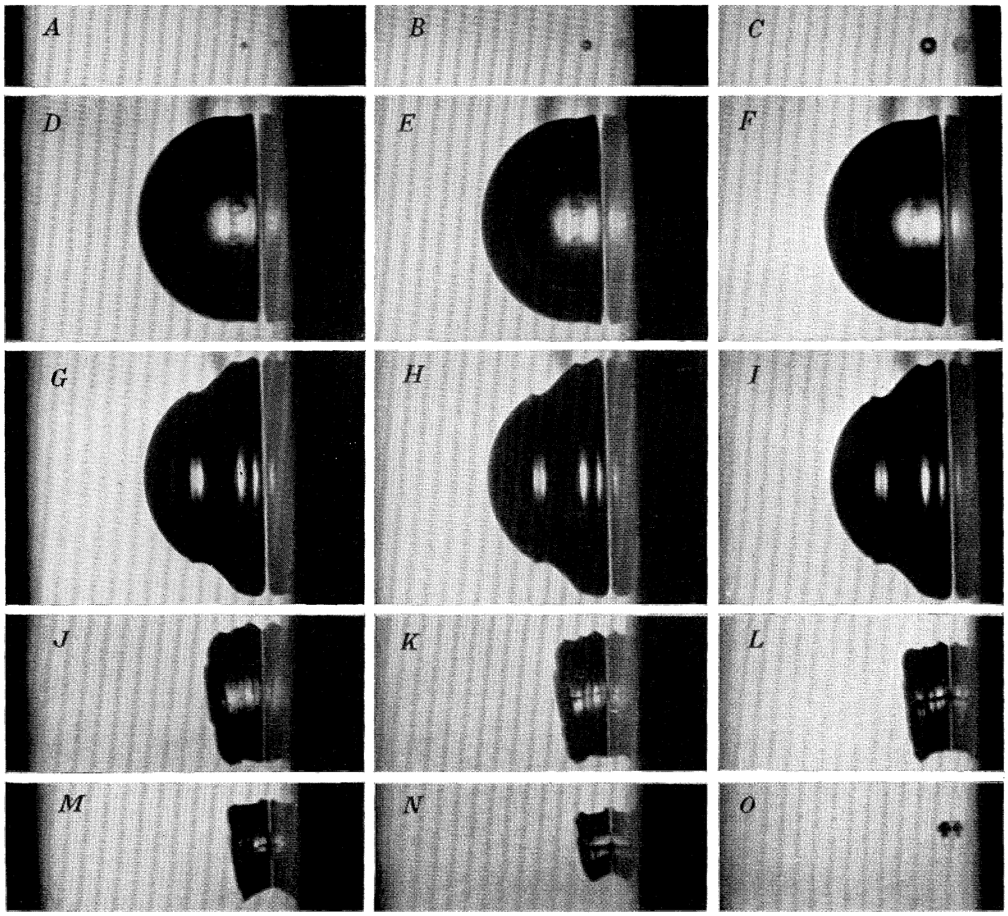

FIGURE 5. Growth of cavity from nucleus very close to a solid wall, and subsequent collapse. Timing: *A*, *B*, *C* at 0, 0·2, 0·4; *D*, *E*, *F* at 5·8, 6·0, 6·2; *G*, *H*, *I* at 11·4, 11·6, 11·8; *J* to *O* at 16·8 to 17·8 ms.

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Appendix

Here the results used in §4 are derived, the situation to which they apply being illustrated in figure 6. An axisymmetric simply connected body (or cavity) displaces a volume $\mathscr{V}$ in a fluid which has constant density ρ and is at rest at infinity. The body moves in the axial direction, measured by the coordinate x, while at the same time it deforms and contracts or dilates. Let $x_0(t)$ denote the position of the centroid of $\mathscr{V}$, and write $\dot{x}_0$ for dx_0/dt.

The fluid being supposed inviscid and its motion therefore irrotational, it follows that the whole motion is determined uniquely at any particular instant, say $t = t_1$, by the normal velocity of the internal boundary (Lamb 1932, §41). The normal velocity may be taken to comprise two parts, the first corresponding to a translation at axial velocity $\dot{x}_0$ of the instantaneous form at $t = t_1$ and the second to the rate of deformation, to represent which the normal velocity *relative to the centroid* is denoted by $\dot{n}$. Accordingly, the velocity potential may be expressed as the sum of two parts,

$$\phi = \dot{x}_0 \phi_1 + \phi_2, \tag{A 1}$$

for which the respective boundary conditions are

$$\frac{\partial \phi_1}{\partial n} = \frac{\partial x}{\partial n}, \quad \frac{\partial \phi_2}{\partial n} = \dot{n} \tag{A 2}$$

over the surface S of the body. (Here n refers to the outward normal.)

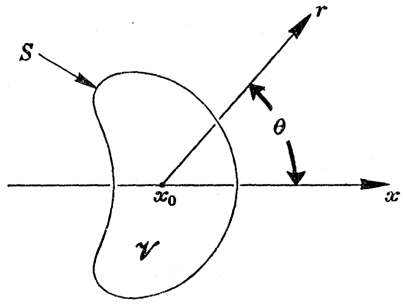

Figure 6. Definition sketch showing axisymmetric body in an infinite fluid.

At large radii r from the centroid, ϕ diminishes at least as rapidly as r^{-1}. Hence the kinetic energy of the fluid is given by the surface integral

$$T = -\tfrac{1}{2}\rho \iint_S \phi \frac{\partial \phi}{\partial n} dS,$$

which, after (A 1) has been substituted, can be put in the form

$$T = \tfrac{1}{2}M\dot{x}_0^2 + J\dot{x}_0 + T', \tag{A 3}$$

with

$$M = -\rho \iint_S \phi_1 \frac{\partial \phi_1}{\partial n} dS, \tag{A 4}$$

$$J = -\tfrac{1}{2}\rho \iint_S \left(\phi_1 \frac{\partial \phi_2}{\partial n} + \phi_2 \frac{\partial \phi_1}{\partial n}\right) dS, \tag{A 5}$$

$$T' = -\tfrac{1}{2}\rho \iint_S \phi_2 \frac{\partial \phi_2}{\partial n} dS. \tag{A 6}$$

We note that, by the definition of ϕ_1, the coefficient M is the 'induced mass' for translations of a *rigid* body formed by freezing the present body in its form at $t = t_1$. Furthermore, the energy T', being independent of $\dot{x}_0$, is a property only of the rate of deformation. Unfortunately for the simplicity of the argument, the coefficient J does not in general vanish. But there are several things to be said about it which were of use in §4.

From Green's second identity applied to the potentials ϕ_1 and ϕ_2, it follows that the two parts of J expressed in (A 5) are equal, so that

$$J = -\rho \iint_S \phi_2 \frac{\partial \phi_1}{\partial n} \, dS$$

$$= -\rho \iint_S \phi_2 \frac{\partial x}{\partial n} \, dS. \tag{A 7}$$

Thus J is completely determined by the deformation relative to the centroid, being independent of the translational motion. Now apply Green's second identity to ϕ_2 and $x - x_0$. It shows that

$$\iint_{S+S'} \phi_2 \frac{\partial x}{\partial n} \, dS = \iint_{S+S'} \frac{\partial \phi_2}{\partial n} (x - x_0) \, dS,$$

where S' is any surface enclosing S. Letting this be a sphere centred on $x = x_0$, we see that both integrals over S' depend only on the coefficient, say A_1, of the dipole term

$$(x - x_0)/r^3 = r^{-2} \cos \theta$$

in the general asymptotic expansion of ϕ_2 for large r. Hence, putting $\partial \phi_2 / \partial n = \dot{n}$ on S, we obtain

$$J = -\rho \left\{ \iint_S \dot{n}(x - x_0) \, dS + 4\pi A_1 \right\} \tag{A 8}$$

(cf. Lamb, §121 *a*). But we have $\iint_S \dot{n}(x - x_0) \, dS = 0$ (A 9)

as a condition that the centroid remains at $x = x_0$ as the body deforms. Thus (A 8) shows J to be proportional to the strength A_1 of the dipole 'far-field' that would be generated by the normal motion $\dot{n}$ of the body in a fluid otherwise at rest.

At first sight one might easily be led into supposing that no dipole far-field could be generated by movements of a body keeping the centroid of the displaced volume fixed, but this conclusion is certainly not true in general. Nevertheless it does appear to be true, roughly speaking, that the body has to develop a large degree of skewness in both form and velocity distribution to make the dipole field—and hence J—a significant factor in the motion as a whole. Another interpretation of J which supports this conclusion intuitively is that, if the motion $\dot{n}$ of the boundary were generated impulsively, J would give the total reaction of the fluid against constraints which kept the centroid from moving in the axial direction. Alternatively, if the body were free to move, J would be interpretable as the Kelvin impulse in an inertial frame of reference moving with the centroid (this interpretation is perhaps obvious from equation (5) in the main text). Thus J is a measure of the tendency of the deformations to self-propel the body in the axial direction, and clearly the body must squirm into a highly skew configuration in order to develop a strong effect of this sort.

If S is a sphere, then J vanishes for all possible velocity distributions $\dot{n}$. This is easily proved by expressing ϕ_2 in a series of solid harmonics and $\dot{n}$ in a corresponding series of surface harmonics (Legendre functions) over the sphere, whereupon (A 9) shows there to be no dipole term in the expansion and the result $J = 0$ follows from (A 8). The same result is found to hold when S is an ellipsoid. Some calculations to examine the magnitude of J in the case of a perturbed sphere have been made by the present authors, expressing the radius of S as an expansion of Legendre functions, but the results are too complicated to justify inclusion here. It will suffice to note that J depends to a first approximation on factors such as $\dot{a}_2 a_3$, where a_2 and a_3 are the coefficients of the second- and third-order Legendre functions.

[The simple case when S remains spherical during the whole motion is worth noting, the result for T having been used by many people in explosion-bubble studies (e.g. by Sir Geoffrey Taylor in a paper included in Hartmann & Hill (1950)). This case gives

$$M = \tfrac{1}{2}\rho\mathscr{V}\dot{x}_0^2 = \tfrac{2}{3}\pi\rho R^3\dot{x}_0^2, \quad J = 0 \quad \text{and} \quad T' = 2\pi\rho R^3\dot{R}^2,$$

where R is the radius of the sphere.]

Suppose that the volume and form of the body are determined by a set of parameters $\lambda_n(t)$ $(n = 0, 1, 2, \ldots)$, which could be, for example, the coefficients of an expansion in Legendre functions. From what has been said it may now be appreciated that T' takes the general form

$$T' = \tfrac{1}{2}\rho \sum_m \sum_n A_{m,n}\dot{\lambda}_m\dot{\lambda}_n, \tag{A 10}$$

in which the $A_{m,n}$ are functions of the λ_n alone; and J has the form expressed in equation (3) of the main text. Also M is a function of the λ_n alone.

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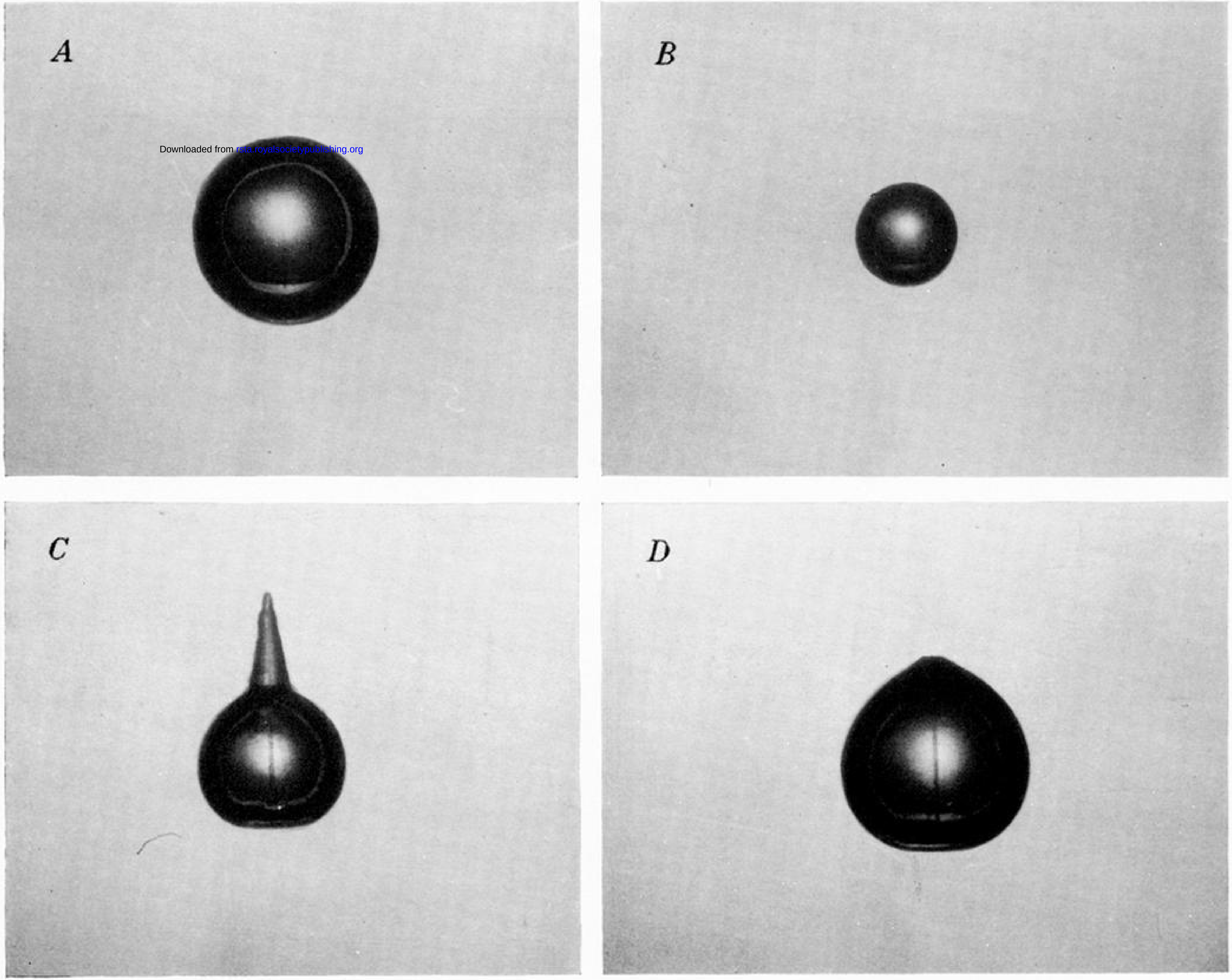

IGURE 2. Photographs taken during collapse (A, B) and rebound (C, D) of cavity far from boundaries of liquid. Interval between frames, 2 ms.

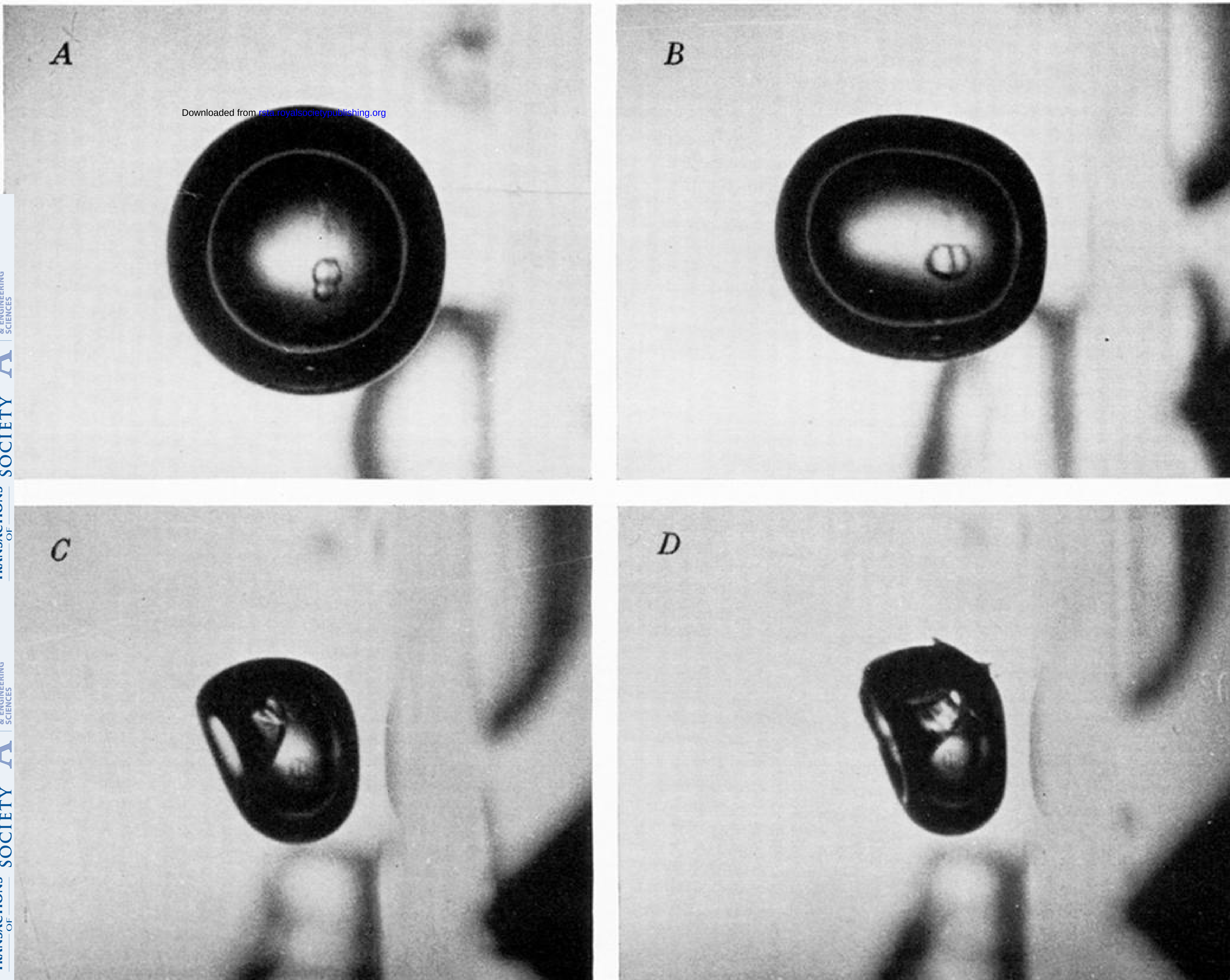

FIGURE 3. Collapse of cavity near a solid wall. Timing: *A*, *B*, *C*, *D* at 0, 5, 9, 10 ms.

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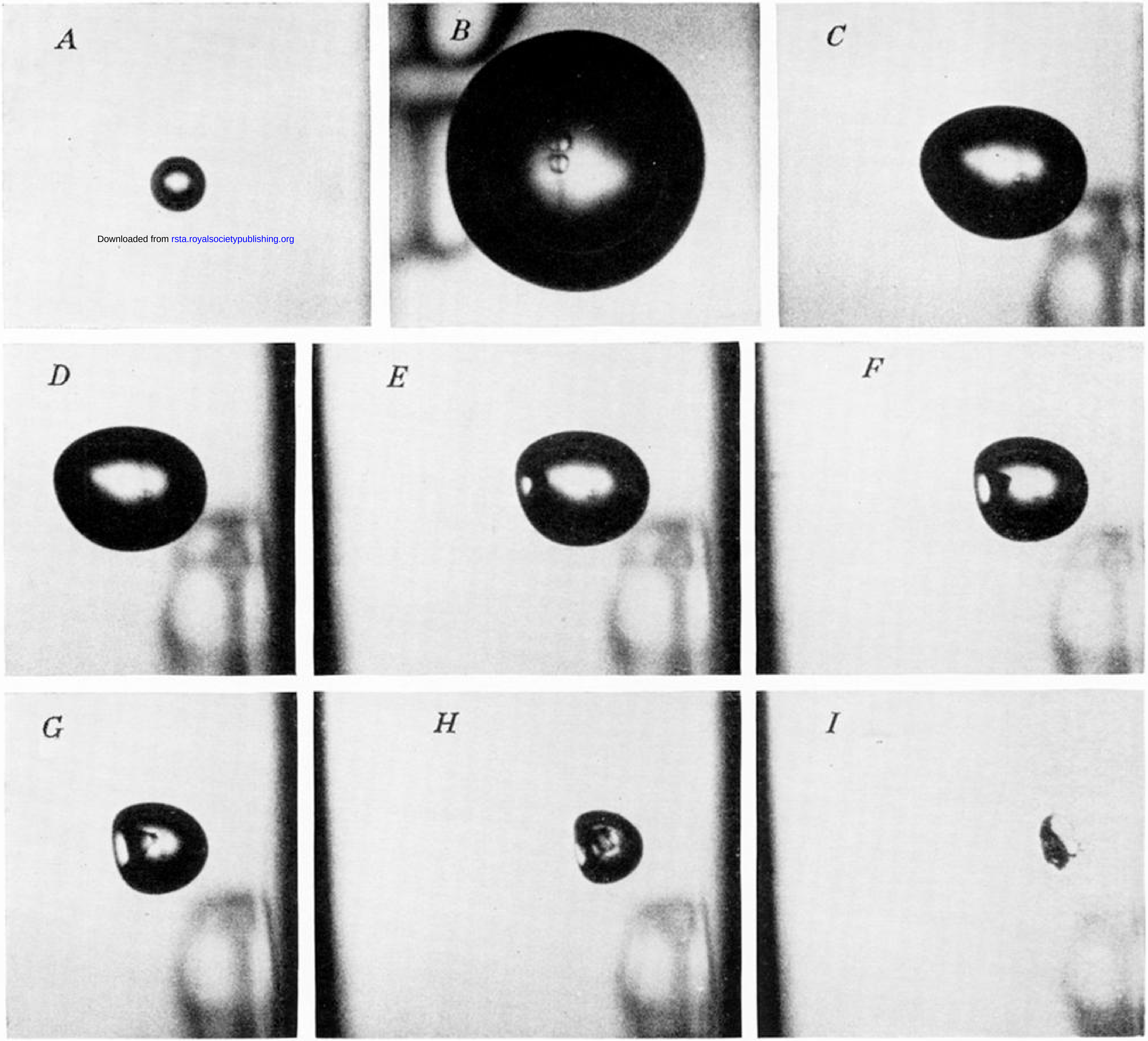

FIGURE 4. Growth and collapse of cavity near a solid wall. Timing: *A*, *B*, *C* at 0, 5·8, 8·8; *D*, *E*, *F* at 9·4, 9·6, 9·8; *G*, *H*, *I* at 10·0, 10·2, 10·4 ms.

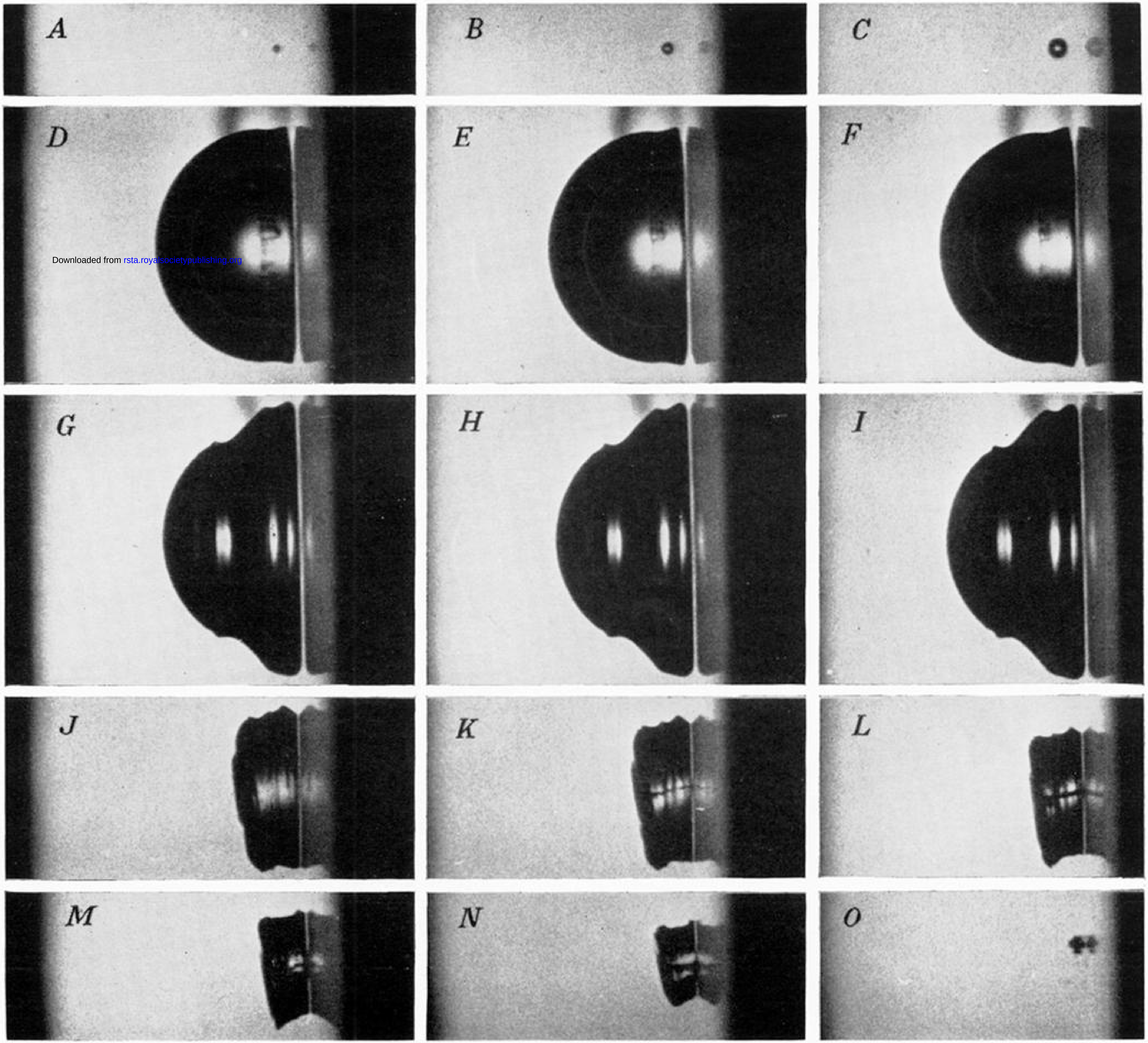

FIGURE 5. Growth of cavity from nucleus very close to a solid wall, and subsequent collapse. Timing: *A*, *B*, *C* at 0, 0·2, 0·4; *D*, *E*, *F* at 5·8, 6·0, 6·2; *G*, *H*, *I* at 11·4, 11·6, 11·8; *J* to *O* at 16·8 to 17·8 ms.